Hawking Radiation of Photons in a Vaidya-de Sitter Black Hole

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Abstract

Hawking evaporation of photons in a Vaidya-de Sitter black hole is investigated by using the method of generalized tortoise coordinate transformation. Both the location and the temperature of the event horizon depend on the time. It is shown that Hawking radiation of photons exists only for the complex Maxwell scalar ϕ_0 in the advanced Eddington-Finkelstein coordinate system. This asymmetry of Hawking radiation for different components of Maxwell fields probably arises from the asymmetry of spacetime in the advanced Eddington-Finkelstein coordinate system. It is shown that the black body radiant spectrum of photons resembles that of Klein-Gordon particles.

Key Words: Hawking radiation, Maxwell equation, Vaidya-type black hole, generalized tortoise coordinate transformation

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1 Introduction

Hawking's investigation of quantum effects [1] interpreted as the emission of a thermal spectrum of particles by a black hole event horizon sets a significant landmark on black hole physics. In the last few decades, much work has been done on the Hawking evaporation of black holes in some spherically symmetric and non-static spacetimes [2, 3]. In a recent paper [4] (here refer to Paper I), we re-examined the Hawking effect of Dirac particles in a Vaidya-type black hole by means of the generalized tortoise transformation method. We considered simultaneously the asymptotic behaviors of the first-order and second-order forms of Dirac equations near the event horizon, and eliminated the crossing-terms of the first-order derivatives in the second-order equations by using the relations between the first-order derivatives of the radial Dirac equations. We showed that the Hawking radiation exists only for P_2, Q_1 components of Dirac spinors in virtue of the restricts imposed by the limiting form of its first-order equations. We conceived that this asymmetry of the Hawking radiation for different spinorial components probably originates from the asymmetry of space-time in the advanced Eddington-Finkelstein coordinate system.

In this paper, we deal with the thermal radiation of photons in a Vaidya-de Sitter space-time. The method used here is the same as that presented in Paper I, namely, we consider the asymptotic behaviors of the first-order and second-order forms of Maxwell equations in the vicinity of the event horizon, and recast each second-order equation to a standard wave equation near the event horizon. The location and the temperature of the event horizon are shown to be dependent on the time. The black body radiation spectrum of photons resembles that of Klein-Gordon scalar particles. We find that due to the restricts put on the Hawking radiation by the limiting form of the first-order Maxwell equations, not all Maxwell complex scalars but ϕ_0 displays the property of thermal radiation. This asymmetry of Hawking radiation for different field components in a Vaidya-type spacetime is thought to be a common feature shared by all particles with higher-spins.

The paper is organized as follows: In Sec. 2, the explicit form of sourceless Maxwell equations within the framework of Newman-Penrose formalism [5] are written out by choosing an appropriate null tetrad in the Vaidya-de Sitter geometry. By using the method of generalized tortoise coordinate transformation, the event horizon equation is extracted

in Sec. 3 from the asymptotic forms of the radial parts of the first-order Maxwell equations near the event horizon. Then the second-order radial equations are manipulated in Sec. 4 by the same procedure and recast into a standard wave equation near the event horizon, in the meanwhile, an exact expression of the "surface gravity" of the event horizon is also obtained by adjusting the parameter κ. Sec. 5 is devoted to deriving the thermal radiation spectrum of photons from the event horizon. Finally we present some discussions in Sec. 6.

2 Sourceless Maxwell Equations

The metric of a Vaidya black hole with a cosmological constant Λ is given in the advanced Eddington-Finkelstein coordinate system by

$$ds^2 = 2dv(Gdv - dr) - r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{1}$$

where $2G = 1 - 2M/r - \Lambda r^4/3$, in which mass $M(v)$ of the hole is a function of the advanced time v.

The geometry of this Petrov type-D space-time is characterized by two kinds of surfaces of particular interest: the apparent horizon $r_{AH} = 2M$ (coincides with the timelike limit surfaces r_{TLS}) and the event horizon $r_{EH} = r_H$. The apparent horizon is the outmost trapped surface, while the event horizon is necessary a null-surface $r = r(v)$ that satisfies the null-surface conditions $g^{ij}\partial_i F \partial_j F = 0$ and $F(v, r) = 0$. Traditionally the latter is calculated approximately by the simple physical condition that the photons at the event horizon are stuck or unaccelerated in the sense that $\ddot{r} \approx 0$. An effective method to determine the location of the event horizon of a dynamic black hole is called as generalized tortoise coordinate transformation (GTCT) with which we can apply it to the null hypersurface equation $g^{ij}\partial_i F \partial_j F = 0$ and then take the limits approaching the event horizon. However in Paper I, the event horizon equation is extracted from the asymptotic forms of the radial parts of the first-order Dirac equations near the event horizon. In this paper, we derive it by the same procedure but with the Maxwell equations in place of the Dirac equation here.

When the back reaction of the massless spin-1 test particles on the background geometry is neglected, the electromagnetic field equation is given by the Maxwell equation

on a fixed spacetime (1). In order to write out its explicit form in the Newman-Penrose [5] formalism, we establish the following complex null tetrad system $\{\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \overline{\boldsymbol{m}}\}$ that satisfies the orthogonal conditions $\boldsymbol{l} \cdot \boldsymbol{n} = -\boldsymbol{m} \cdot \overline{\boldsymbol{m}} = 1$. Thus the covariant one-forms can be written as

$$\begin{aligned}
\boldsymbol{l} &= dv\,, & \boldsymbol{m} &= \frac{-r}{\sqrt{2}}\big(d\theta + i\sin\theta d\varphi\big)\,,\\
\boldsymbol{n} &= Gdv - dr\,, & \overline{\boldsymbol{m}} &= \frac{-r}{\sqrt{2}}\big(d\theta - i\sin\theta d\varphi\big)\,,
\end{aligned} \tag{2}$$

and their corresponding directional derivatives are

$$\begin{aligned}
D &= -\frac{\partial}{\partial r}\,, & \delta &= \frac{1}{\sqrt{2}r}\Big(\frac{\partial}{\partial\theta} + \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\Big)\,,\\
\Delta &= \frac{\partial}{\partial v} + G\frac{\partial}{\partial r}\,, & \overline{\delta} &= \frac{1}{\sqrt{2}r}\Big(\frac{\partial}{\partial\theta} - \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\Big)\,.
\end{aligned} \tag{3}$$

It is not difficult to determine the non-vanishing Newman-Penrose complex spin coefficients [5] in the above null-tetrad as

$$\begin{aligned}
\rho &= 1/r\,, & \gamma &= -G_{,r}/2 = -dG/2dr\,,\\
\mu &= G/r\,, & \beta &= -\alpha = \cot\theta/(2\sqrt{2}r)\,.
\end{aligned} \tag{4}$$

Inserting for the appropriate spin coefficients into the sourceless Maxwell equations [5, 6] in the Newman-Penrose formalism [5]

$$\begin{aligned}
&(D - 2\rho)\phi_1 - (\overline{\delta} + \tilde{\pi} - 2\alpha)\phi_0 = -\tilde{\kappa}\phi_2\,,\\
&(\delta - 2\tau)\phi_1 - (\Delta + \mu - 2\gamma)\phi_0 = -\sigma\phi_2\,,\\
&(D + 2\epsilon - \rho)\phi_2 - (\overline{\delta} + 2\tilde{\pi})\phi_1 = -\tilde{\lambda}\phi_0\,,\\
&(\delta + 2\beta - \tau)\phi_2 - (\Delta + 2\mu)\phi_1 = -\tilde{\nu}\phi_0\,,
\end{aligned} \tag{5}$$

we obtain

$$\begin{aligned}
&\Big(\frac{\partial}{\partial r} + \frac{2}{r}\Big)\phi_1 + \frac{1}{\sqrt{2}r}\mathcal{L}_1\phi_0 = 0\,,\\
&\Big(\mathcal{D} + \frac{G}{r} + G_{,r}\Big)\phi_0 - \frac{1}{\sqrt{2}r}\mathcal{L}^{\dagger}{}_0\phi_1 = 0\,,\\
&\Big(\frac{\partial}{\partial r} + \frac{1}{r}\Big)\phi_2 + \frac{1}{\sqrt{2}\rho}\mathcal{L}_0\phi_1 = 0\,,\\
&\Big(\mathcal{D} + \frac{2G}{r}\Big)\phi_1 - \frac{1}{\sqrt{2}\rho^*}\mathcal{L}^{\dagger}{}_1\phi_2 = 0\,,
\end{aligned} \tag{6}$$

here we have defined operators

$$\mathcal{D} = \frac{\partial}{\partial v} + G\frac{\partial}{\partial r}\,,$$
$$\mathcal{L}_n = \frac{\partial}{\partial\theta} + n\cot\theta - \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\,,$$
$$\mathcal{L}^{\dagger}{}_n = \frac{\partial}{\partial\theta} + n\cot\theta + \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\,.$$

By substituting $\Phi_0 = r\phi_0$, $\Phi_1 = \sqrt{2}r^2\phi_1$, $\Phi_2 = r\phi_2$ into Eq. (6), we have

$$\begin{aligned}
&\frac{\partial}{\partial r}\Phi_1 + \mathcal{L}_1\Phi_0 = 0\,, \qquad && 2r^2\Big(\mathcal{D} + G_{,r}\Big)\Phi_0 - \mathcal{L}^{\dagger}{}_0\Phi_1 = 0\,,\\
&2r^2\frac{\partial}{\partial r}\Phi_2 + \mathcal{L}_0\Phi_1 = 0\,, && \mathcal{D}\Phi_1 - \mathcal{L}^{\dagger}{}_1\Phi_2 = 0\,.
\end{aligned} \tag{7}$$

3 Event Horizon

Eq. (7) can be decoupled as

$$\Phi_0 = R_0(v,r)S_0(\theta,\varphi)\,, \quad \Phi_1 = R_1(v,r)S_1(\theta,\varphi)\,, \quad \Phi_2 = R_2(v,r)S_2(\theta,\varphi)$$

to the radial part

$$\begin{aligned}
&\frac{\partial}{\partial r}R_1 + \lambda R_0 = 0\,, \qquad && 2r^2\Big(\mathcal{D} + G_{,r}\Big)R_0 + \lambda R_1 = 0\,,\\
&2r^2\frac{\partial}{\partial r}R_2 + \lambda R_1 = 0\,, && \mathcal{D}R_1 + \lambda R_2 = 0\,,
\end{aligned} \tag{8}$$

and the angular part

$$\begin{aligned}
&\mathcal{L}_1 S_0 = \lambda S_1\,, \qquad && \mathcal{L}^{\dagger}{}_0 S_1 = -\lambda S_0\,,\\
&\mathcal{L}_0 S_1 = \lambda S_2\,, && \mathcal{L}^{\dagger}{}_1 S_2 = -\lambda S_1\,,
\end{aligned} \tag{9}$$

where $\lambda = \sqrt{\ell(\ell+1)}$ is a separation constant. All functions $S_0(\theta,\varphi)$, $S_1(\theta,\varphi)$ and $S_2(\theta,\varphi)$ are, respectively, spin-weighted spherical harmonics ${}_pY_{\ell m}(\theta,\varphi)$ with spin-weight $p = 1, 0, -1$, satisfying the following equation [7]

$$\begin{aligned}
\Big[&\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{2ip\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi}\\
&- p^2\cot^2\theta + p + (\ell-p)(\ell+p+1)\Big]{}_pY_{\ell m}(\theta,\varphi) = 0\,.
\end{aligned} \tag{10}$$

As to the Hawking radiation, we should be concerned about the asymptotic behaviors of the radial part of Eq. (8) in the vicinity of the event horizon. Because the Vaidya-de Sitter spacetime is spherically symmetric, we can introduce as a working ansatz the following GTCT as did in the Paper I

$$r_* = r + \frac{1}{2\kappa}\ln[r - r_H(v)]\,, \quad v_* = v - v_0\,, \tag{11}$$

where $r_H = r(v)$ is the location of the event horizon, and κ is an adjustable parameter and is unchanged under the tortoise transformation. The parameter v_0 is an arbitrary constant which characterizes the initial instant of the hole. From formula (11), we can deduce some useful relations for the derivatives as follows:

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r_*} + \frac{1}{2\kappa(r - r_H)}\frac{\partial}{\partial r_*}\,, \quad \frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} - \frac{\dot{r}_H}{2\kappa(r - r_H)}\frac{\partial}{\partial r_*}\,.$$

The quantities $\dot{r}_H = \partial r_H/\partial v$ is the rate of the event horizon varying in time v. It describes the evolution of the black hole event horizon in the time, which reflects the presence of quantum ergosphere near the event horizon.

Now let us consider first the asymptotic behaviors of Eq. (8) near the event horizon. Under the transformations (11), Eq. (8) can be reduced to the following forms

$$\begin{aligned} &\frac{\partial}{\partial r_*}R_1 = 0\,, \qquad 2r_H^2(\dot{r}_H - G)\frac{\partial}{\partial r_*}R_0 = 0\,, \\ &2r_H^2\frac{\partial}{\partial r_*}R_2 = 0\,, \qquad (\dot{r}_H - G)\frac{\partial}{\partial r_*}R_1 = 0\,, \end{aligned} \tag{12}$$

after being taken the $r \to r_H(v_0)$ and $v \to v_0$ limits.

From Eq. (12), we know that $R_1(r_*)$ and $R_2(r_*)$ are regular on the event horizon,

$$\frac{\partial}{\partial r_*}R_1 = \frac{\partial}{\partial r_*}R_2 = 0\,, \tag{13}$$

thus a reasonable solution to Eq. (12) is that the derivatives $\frac{\partial}{\partial r_*}R_0$ does not vanish. The sole possibility we are left for the existence of a nontrial solution of R_0 is (as for $r_H \neq 0$)

$$2G(r_H) - 2\dot{r}_H = 0\,, \tag{14}$$

which determines the location of the event horizon. It is interesting to note that the event horizon equation (14) coincides with that inferred from the null surface equation $g^{ij}\partial_i F \partial_j F = 0$. Because r_H depends on time v, the location of the event horizon and the shape of the black hole change with time.

4 Hawking Temperature

In the preceding section, we have deduced the event horizon equation from the limiting form of the separated radial part of the first-order Maxwell equations. Applying a similar procedure to its second-order forms, we can derive the Hawking temperature and the thermal radiation spectrum. A straightforward calculation gives the second-order radial equations

$$2r^2\Big[G\frac{\partial^2}{\partial r^2}+\frac{\partial^2}{\partial v\partial r}+2(G_{,r}+\frac{G}{r})\frac{\partial}{\partial r}+\frac{2}{r}\frac{\partial}{\partial v}+\frac{2G_{,r}}{r}+G_{,rr}\Big]R_1-\lambda^2R_1=0\,,\tag{15}$$

$$2r^2\Big(G\frac{\partial^2}{\partial r^2}+\frac{\partial^2}{\partial v\partial r}+G_{,r}\frac{\partial}{\partial r}\Big)R_1-\lambda^2R_1=0\,,\tag{16}$$

$$2r^2\Big(G\frac{\partial^2}{\partial r^2}+\frac{\partial^2}{\partial v\partial r}+\frac{2G}{r}\frac{\partial}{\partial r}\Big)R_2-\lambda^2R_2=0\,.\tag{17}$$

Given the GTCT in Eq. (11) and after some tedious calculations, Eqs. (15-17) have the following limiting forms near the event horizon $r=r_H$

$$\begin{aligned}&\Big\{\Big[\frac{A}{2\kappa}+4G(r_H)-2\dot{r}_H\Big]\frac{\partial^2}{\partial r_*^2}+2\frac{\partial^2}{\partial r_*\partial v_*}+\Big[-A+4G_{,r}(r_H)\\&+\frac{4G(r_H)-4\dot{r}_H}{r_H}\Big]\frac{\partial}{\partial r_*}\Big\}R_0=\Big\{\Big(\frac{A}{2\kappa}-2\dot{r}_H\Big)\frac{\partial^2}{\partial r_*^2}+2\frac{\partial^2}{\partial r_*\partial v_*}\\&+\Big[-A+4G_{,r}(r_H)\Big]\frac{\partial}{\partial r_*}\Big\}R_0=0\,,\end{aligned}\tag{18}$$

$$\begin{aligned}&\Big\{\Big[\frac{A}{2\kappa}+4G(r_H)-2\dot{r}_H\Big]\frac{\partial^2}{\partial r_*^2}+2\frac{\partial^2}{\partial r_*\partial v_*}+[-A+2G_{,r}(r_H)]\frac{\partial}{\partial r_*}\Big\}R_1\\&=\Big[\Big(\frac{A}{2\kappa}-2\dot{r}_H\Big)\frac{\partial^2}{\partial r_*^2}+2\frac{\partial^2}{\partial r_*\partial v_*}\Big]R_1=0\,,\end{aligned}\tag{19}$$

$$\begin{aligned}&\Big\{\Big[\frac{A}{2\kappa}+4G(r_H)-2\dot{r}_H\Big]\frac{\partial^2}{\partial r_*^2}+2\frac{\partial^2}{\partial r_*\partial v_*}+\Big[-A+\frac{4G(r_H)}{r_H}\Big]\frac{\partial}{\partial r_*}\Big\}R_2\\&=\Big[\Big(\frac{A}{2\kappa}-2\dot{r}_H\Big)\frac{\partial^2}{\partial r_*^2}+2\frac{\partial^2}{\partial r_*\partial v_*}\Big]R_2=0\,,\end{aligned}\tag{20}$$

when r approaches $r_H(v_0)$ and v goes to v_0. In the above, we have used relations $2G(r_H)=2\dot{r}_H$ and $\frac{\partial}{\partial r_*}R_1=\frac{\partial}{\partial r_*}R_2=0$. The coefficient A is an infinite form of 0/0-type with a finite result treated by using of the L' Hôspital rule,

$$A=\lim_{r\to r_H(v_0)}\frac{2G-2\dot{r}_H}{r-r_H}=2G_{,r}(r_H)\,.$$

In order to recast Eqs. (18), (19) and (20) into a standard wave equation near the event horizon, we select the adjustable parameter κ in them such that it satisfies

$$\frac{A}{2\kappa} + 2G(r_H) = \frac{G_{,r}(r_H)}{\kappa} + 2\dot{r}_H \equiv 1\,, \tag{21}$$

which means the "surface gravity" of the horizon is

$$\kappa = \frac{G_{,r}(r_H)}{1 - 2G(r_H)} = \frac{G_{,r}(r_H)}{1 - 2\dot{r}_H}\,, \tag{22}$$

where we have used Eq. (14).

With such a parameter adjustment, these wave equations (18-20) can be reduced to

$$\Big(\frac{\partial^2}{\partial r_*^2} + 2\frac{\partial^2}{\partial r_* \partial v_*} + 2C\frac{\partial}{\partial r_*}\Big)R_0 = 0\,, \tag{23}$$

$$\Big(\frac{\partial^2}{\partial r_*^2} + 2\frac{\partial^2}{\partial r_* \partial v_*}\Big)R_1 = 0\,, \tag{24}$$

$$\Big(\frac{\partial^2}{\partial r_*^2} + 2\frac{\partial^2}{\partial r_* \partial v_*}\Big)R_2 = 0\,, \tag{25}$$

where $C = G_{,r}(r_H)$. Eqs. (23-25) have a standard form of wave equation in the vicinity of the event horizon. We point out that the above parameter adjustment is an important step in our discussions.

5 Thermal Radiation Spectrum

Combining Eqs. (24, 25) with $\frac{\partial}{\partial r_*}R_1 = \frac{\partial}{\partial r_*}R_2 = 0$, we know that R_1 and R_2 are independent of r_* near the event horizon. The solutions $R_1 \sim e^{-i\omega v_*}$ and $R_2 \sim e^{-i\omega v_*}$ indicate that Hawking radiation does not exist for Φ_1 and Φ_2.

Now separating variables to Eq. (23) as $R_0 = R_0(r_*)e^{-i\omega v_*}$, one gets

$$R_0'' = 2(i\omega - C)R_2'\,, \qquad R_0 = c_1 e^{2(i\omega - C)r_*} + c_2\,. \tag{26}$$

The incoming wave solution and the outgoing wave solution to Eq. (23) are, respectively,

$$\begin{aligned} &R_0^{\rm in} \sim e^{-i\omega v_*}\,, \\ &R_0^{\rm out} \sim e^{-i\omega v_*} e^{2(i\omega - C)r_*}\,, \qquad (r > r_H)\,. \end{aligned} \tag{27}$$

Near the event horizon, we have $r_* \sim \frac{1}{2\kappa}\ln(r - r_H)$. Clearly, the outgoing wave $R_0^{\rm out}(r > r_H)$ is not analytic at the event horizon $r = r_H$, but can be analytically extended from

the exterior of the hole into the interior of the hole by the lower complex r-plane

$$(r - r_H) \rightarrow (r_H - r)e^{-i\pi}$$

to

$$\widetilde{R_0^{\rm out}} = e^{-i\omega v_*} e^{2(i\omega - C)r_*} e^{i\pi C/\kappa} e^{\pi\omega/\kappa} \,, \qquad (r < r_H) \,. \tag{28}$$

Following the method of Damour-Ruffini-Sannan's [8], the relative scattering probability of the outgoing wave at the event horizon horizon and the thermal radiation spectrum of photons from the event horizon of the black hole are easily obtained

$$\Big|\frac{R_0^{\rm out}}{\widetilde{R_0^{\rm out}}}\Big|^2 = e^{-2\pi\omega/\kappa} \,, \qquad \langle \mathcal{N}_\omega \rangle \sim \frac{1}{e^{\omega/T} - 1} \,, \tag{29}$$

in which m is the azimuthal quantum number, and the Hawking temperature is

$$T = \frac{\kappa}{2\pi} = \frac{1}{4\pi r_H} \cdot \frac{M r_H - \Lambda r_H^4/3}{M r_H - \Lambda r_H^4/6} \,. \tag{30}$$

The thermal radiation spectrum (29) due to the Bose-Einstein statistics of photons shows that the black hole emits radiation just like a black body emitting scalar particles. The temperature depends on the time and is consistent with that derived from the investigation of the thermal radiation of Dirac particles in a Vaidya-de Sitter black hole [4] with a vanishing electric charge ($Q = 0$).

6 Conclusions

In this paper, we have studied the Hawking radiation of photons in a Vaidya-de Sitter black hole. The location and the temperature of the event horizon given by Equations (14) and (22), respectively, depend on the advanced time v. They can recover the well-known results previously derived in the discussion on the Hawking evaporation of Klein-Gordon and Dirac particles in the same spacetime. Eq. (29) shows that the thermal radiation spectra of photons have the same form as that of Klein-Gordon particles in a Vaidya-type black hole with a cosmological constant Λ.

In summary, we have dealt with the asymptotic behaviors of the separated radial equations near the event horizon, not only its first-order form but also its second-order form. We find that the limiting form of its first-order equations puts very strong restrict on the Hawking effect, that is, not all components of Maxwell complex scalars but ϕ_0

displays the property of thermal radiation. As is revealed in Paper I, we argued that this asymmetry of Hawking radiation for different components of Maxwell fields probably stem from the asymmetry of spacetimes in the advanced Eddington-Finkelstein coordinate. We think this is a common character shared by the thermal radiation of particles with higher spins in any Vaidya-type spherically symmetric black hole.

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